

Applying Axiomatic Design to Risk Assessment

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Abstract. As the complexity of our society increase, managing risk is an important task in maintaining healthy and sustainable societies. Risk assessment is a widely used method to identify events associated with high probability of occurrence and severe consequences. Although simple and easy to understand, risk assessment is over-simplified with a single estimate of occurrence probability and a single severity evaluation of each event. Axiomatic design (AD) decomposes an overall functional requirement (FR) into a set of element FRs. Although an uncoupled design is ideal with each FR mapping to a single design parameter (DP), a design, in many cases, is coupled and one or more single FRs may depend on multiple DPs. This study shows how we can apply the design equation formulation of AD to risk assessment by identifying the failure probability of each DP, estimating the severity of not meeting each FR, and then calculating the risk associated with each FR. The method identifies improvement on which DP is most effective in reducing the overall system risk. Our analysis also shows how unwanted design interference can cause unexpected serious negative consequences. The designer is encouraged to apply AD to recognize design interference and spend efforts in removing them before production.

Keywords: Risk assessment, Axiomatic Design, Design record graph.

1 Introduction

1.1 A Subsection Sample

Risk assessment [1] is a popular tool for managers and business groups for identifying risks that exist in their business environment with probabilities of occurrence that are too large with severe consequences. Risk is typically calculated with the equation:

$$(\text{Risk}) = (\text{Probability of occurrence}) \times (\text{Severity of consequence}) \quad (1)$$

without definite rules for what numbers to give to the two terms in the right-hand side of the equation. Thus, there is no established quantification guidelines for risk.

A risk assessment session often proceeds with a table on the side to aid the person or group of people performing the task (performer). Fig. 1 shows a typical risk assessment table.

		Likelihood			
		Very Likely	Likely	Unlikely	Very Unlikely
Consequence	Fatal	Critical	Critical	High	Moderate
	Major Injury	Critical	High	Moderate	Low
	Minor Injury	High	Moderate	Low	Low
	Injury Negligible	Moderate	Low	Low	Very Low

Fig. 1. Risk Assessment Table

With this table on the side, the performer lists out foreseeable events that are unwanted. The performer then makes two independent evaluations for each event; one is how likely the event takes place, and the other, what are the consequences of the event. These evaluations have no rules for making them and thus, relies on the gut feeling of the performer.

If an event is “Very Likely” to happen and with the consequence of “Fatality,” the risk assessment table says that the event is critical, and the business must take some measures to lower the likelihood of the event happening or lessen the severity of the consequence before proceeding further with their operations. Even if the likelihood is only “Likely,” the table stills says that it is critical. In contrast, if the event leads only to “Minor Injuries” with a likelihood of “Unlikely,” the event has low priority in terms of requiring some actions.

Performing risk assessment, therefore, is simple and easy for those maybe not so well equipped with mathematical proficiency in linear algebra. It yet gives a nice visual representation of analysis using a table. In fact, however, performing risk assessment is far better than not carrying out any risk analysis. Note for this analysis, that each unwanted event is handled independently. It proceeds with an implicit assumption that event A has no effect on the likelihood of event B.

This table is often called “risk assessment matrix” from its regular appearance with all cells filled with quantities. Some practices of risk assessment assign numbers to likelihood and consequence and each cell shows the result of multiplying the two evaluations. If the likelihood and consequence quantifiers are larger with higher probability of occurrence and more serious outcome, respectively, larger products in the cells indicate events that businesses have to act on.

This risk assessment table showing the level of risk for each event, however, is not like a matrix used in linear algebra. Such mathematical matrices indicate mapping from one linear space to another. In terms of Axiomatic Design (AD) [2], the matrix that maps a design parameter (DP) to a functional requirement (FR) vector is called the design matrix (DM) and AD will examine a DM to tell the quality of the design it represents. The first axiom of AD drives designs to be uncoupled, i.e., each FR is realized by a single DP and the other DPs has no effect on the performance of FR. In this case, the DM is diagonal like the following Equation (2) shows.

$$\begin{Bmatrix} FR_1 \\ FR_2 \\ \vdots \\ FR_n \end{Bmatrix} = \begin{bmatrix} X & 0 & \cdots & 0 \\ 0 & X & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \\ \vdots \\ DP_n \end{Bmatrix} \quad (2)$$

The cells in a risk assessment table are independent from each other. If we look at the quantities, or levels in a single row or column they increase or decrease monotonically simply because the headers likelihood and consequence are arranged monotonic. A quantity or level in a cell has nothing to do with those in the adjacent ones.

From its nature of evaluating each event independently, risk assessment does not evaluate effects of an event on others. Many risk assessments evaluate natural hazard events like earthquake, tsunami, storms, or fires, or risks with our daily work like, falling from a ladder, power outages, or a sudden leave or sickness of a worker. Evaluating levels of consequences of these events independently is somewhat acceptable, however, we know that many unwanted events can trigger occurrence of other events, like an earthquake causing tsunami or sickness leading to the worker falling from a ladder.

This paper proposes a more rigorous approach to risk assessment by applying design equation (DE) from AD to correlate DPs to FRs. Our method starts with a design equation with DPs having their probabilities of failure and the DE leads to finding likelihood of each FR failing. Each FR has its own severity of consequence and evaluating this consequence remains the same with conventional risk assessment. In other words, this paper shows a systematic approach to evaluate probability of a FR failure using AD.

2 Smart phone design and its risk assessment

2.1 Smart phone design

To explain the concept of applying AD to risk assessment, we discuss an existing design of a smart phone and how we propose analyzing the risks associated with the FRs they have.

Fig. 2 shows the back and front of a smart phone (iPhone 11) with its parts identified [3]. Some of the parts inside are guessed, and probably there are more parts, especially IC Chips for purposes.

Fig. 3 shows the DPs we identified for our AD analysis. Note that we bundled CPU, Memory, AD converter, and DA converter into PC board. The DP nodes with dark gray indicate parts that are on the rear side or inside the unit and are invisible from the front. The DP nodes with light gray color are parts arranged on the sides.



Fig. 2. iPhone 11 and its parts

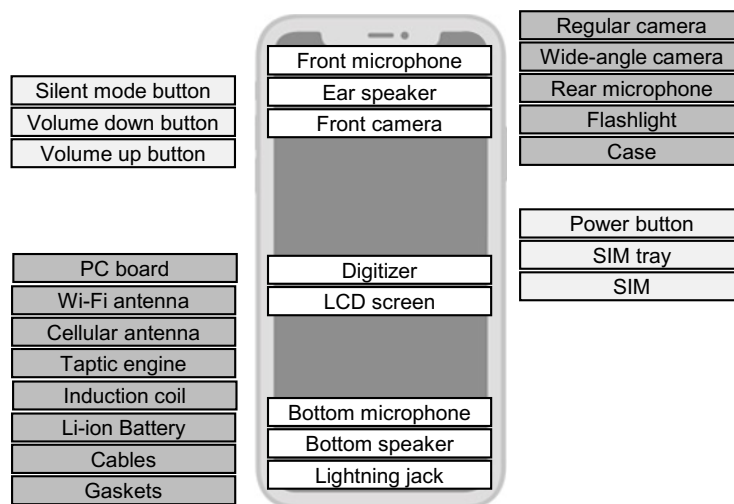


Fig. 3. DPs of iPhone 11 in our analysis

2.2 Design Record Graph

A Design Record Graph (DRG) [4] starts with the overall FR for a product, iteratively subdivides the FR into smaller FR until a set of FRs that are no longer practical for further subdivision. The elements in this FR set are called functional elements (FEs). Then an FE maps to one or more physical elements (PEs). Multiple PEs bundle to define small assemblies which next combine into larger assemblies. The binding continues until the product is defined. Fig. 4 shows the DRG for iPhone 11 in Fig. 3.

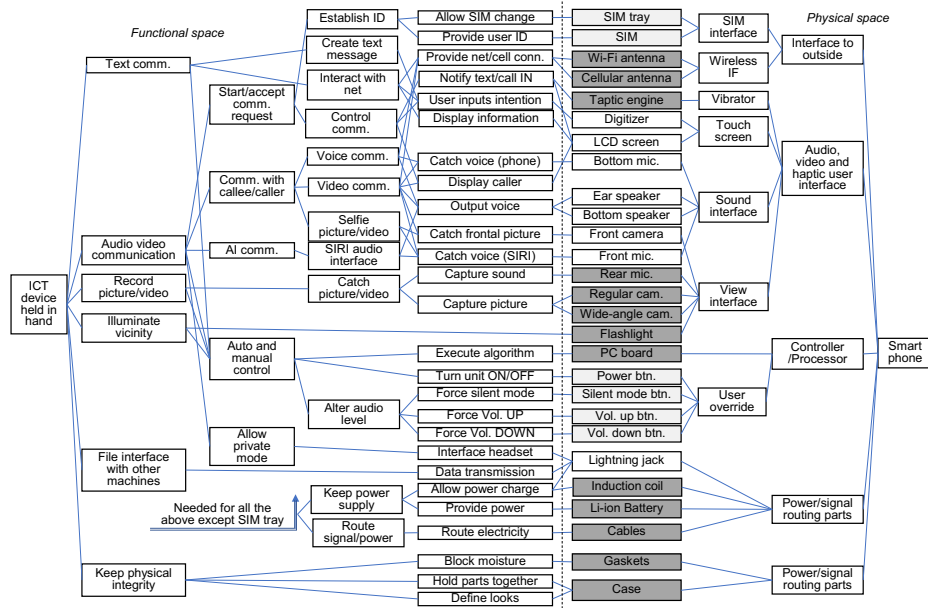


Fig. 4. DRG of iPhone 11

In the next subsection 2.3, we map this DRG into a DE in AD. AD's first Independence Axiom tells the designer to keep independence of FRs [2]. Ideally one FR maps to a single DP. When we interpret this axiom to DRG, it means to have a ladder like mapping from functional space to physical space. Our experience with novice designers like graduate school students, suggests that working with the graphical interface of a DRG to establish a ladder like mapping is easier than to work directly with the DM. Fig. 5 takes the third to the ninth elemental FRs from the top of Fig. google 4 and their corresponding 8 elemental DPs and works out FRs rephrasing to reach a one-to-one correspondence.

Axiom 2, the Independence Axiom of AD, guides the designer in reaching a better design by aiming at minimizing the information content of the design when there are multiple design options. This approach minimizes the risk in not meeting the desired FRs [5]. In our paper, we show how risks of DPs affect the FRs with AD and we reach a maintenance scheme to turn unacceptable risks with FRs into reasonable ranges.

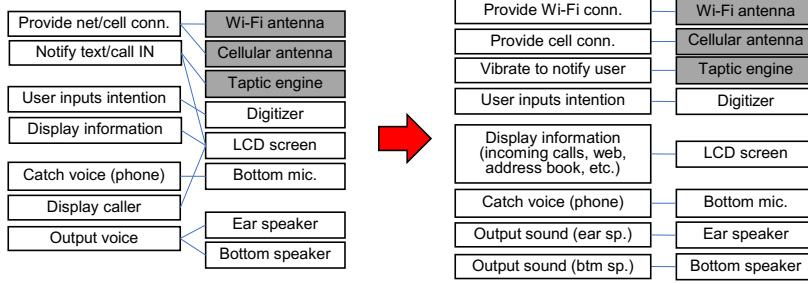


Fig. 5. Rephrasing FRs for independence

2.3 Design Equation

FEs in a DRG correspond to FR elements in AD, and PEs to DP elements. We can then write the DE for an iPhone 11 as follows:

$$\left\{ \begin{array}{l}
 \text{Allow SIM change} \\
 \text{Provide user ID} \\
 \text{Provide net/cell conn.} \\
 \text{Notify text/call IN} \\
 \text{User inputs intention} \\
 \text{Display information} \\
 \text{Display caller} \\
 \text{Catch voice (phone)} \\
 \text{Output voice} \\
 \text{Catch front picture} \\
 \text{Catch voice (SIRI)} \\
 \text{Capture sound} \\
 \text{Capture picture} \\
 \text{Illuminate vicinity} \\
 \text{Execute algorithm} \\
 \text{Turn unit ON/OFF} \\
 \text{Force silent mode} \\
 \text{Force Vol UP} \\
 \text{Force Vol DOWN} \\
 \text{Interface headset} \\
 \text{Data transmission} \\
 \text{Allow power charge} \\
 \text{Provide power} \\
 \text{Route electricity} \\
 \text{Block moisture} \\
 \text{Hold parts together} \\
 \text{Define looks}
 \end{array} \right\} = \mathbf{DM} \left\{ \begin{array}{l}
 \text{SIM tray} \\
 \text{SIM} \\
 \text{Wi-Fi antenna} \\
 \text{Cellular antenna} \\
 \text{Taptic engine} \\
 \text{Digitizer} \\
 \text{LCD screen} \\
 \text{Bottom mic.} \\
 \text{Ear speaker} \\
 \text{Bottom speaker} \\
 \text{Front camera} \\
 \text{Front mic} \\
 \text{Rear mic} \\
 \text{Regular cam} \\
 \text{Wide-angle cam} \\
 \text{Flashlight} \\
 \text{PC board} \\
 \text{Power btn} \\
 \text{Silent mode btn} \\
 \text{Vol.up btn} \\
 \text{Vol.down btn} \\
 \text{Lightning jack} \\
 \text{Li-ion Battery} \\
 \text{Cables} \\
 \text{Gaskets} \\
 \text{Case}
 \end{array} \right\} \quad (3)$$

where equation (4) below shows the elements of DM in equation (3).

The matrix in equation (4) has its non-zero elements in white background so they are easier to identify among the zero elements shaded in gray. Otherwise, the matrix would appear like a quiz of finding non-zero entries among all the zeros. Note that the DM in equation (4) is not diagonal. We produced this DM for Fig. 4 instead of working in the independence FR rephrasing in Fig. 5. Pursuing an uncoupled design, however, is not the point of this study.

A diagram of a 10-lane PCB connector layout. It shows a central row of ten vertical yellow bars representing lanes, enclosed in a white rounded rectangle. Red arrows point from labels to specific lanes: 'Lane 1 negative' to the 1st lane, 'Lane 1 positive' to the 2nd lane, 'ID/control 1' to the 3rd lane, 'Power' to the 4th lane, 'ID/control 0' to the 5th lane, 'Lane 0 negative' to the 6th lane, 'Lane 0 positive' to the 7th lane, and 'Ground' to the 10th lane. The entire assembly is within a grey rounded rectangle.

Fig. 6. Configuration of 8 pins with a lightning connector

Separating out DPs of the lightning jack for its three functions is difficult for the following reason. A lightning connector has 8 pins as Fig. 6 shows [6]. The pin configuration on the other side is in a flipped sequence so that you do not have to worry about connecting a lightning jack and a connector in a reverse manner.

When used for charging the Lithium-ion battery, we first thought only the two pins, Power and Ground, are in use. When we measured lightning cable pins connected to a USB AC power adaptor, the Power pin measured 5V relative to the Ground pin, however, the Lane 0 positive pin also measured 1V relative to the Ground pin. Maybe, it is the way of telling the lightning jack that the user intends to charge the battery. All other 5 pins measured 0V relative to the Ground pin. When in other modes of transferring signals for the headset or data communication, the Ground pin must be used for identifying status of the other signal pins. If we want to define DPs for each FR mode, like in Fig. 5, we will define different DPs of “Lightning jack in charge mode,” “Lightning jack in sound communication mode,” and “Lightning jack in data transfer mode.” But again, aiming for an uncoupled design is not the point of this study.

Note that the non-zero elements of the DM in equation (4) are 1, 3 or 9. We followed the convention used in Reference [7]. Directly quoting the quantification scheme for quantities 1, 3, and 9 with an additional condition for assigning 0,

- 0= no link: the function is not affected at all failing contribution of the component;
- 1= weak link: the function is lightly degraded failing contribution of the component;
- 3= middle link: the function is only reduced failing contribution of the component;
- 9= strong link: the function is completely annulled failing contribution of the component;

Which quantity to assign to each FR-DP relation was at the discretion of the authors.

3 Risk analysis

3.1 Failure probabilities of DPs

For our risk analysis, we start with estimating the failure frequencies of the DPs. The quantity we adopt is the annual failure rate (AFR), i.e., average number of failures per year [8]. It relates to mean time before failure (MTBF) as follows [8]:

$$\text{AFR} = 1/\text{MTBFyears} = 8760/\text{MTBFhours} \quad (5)$$

Table 1 shows the AFR values we assigned to our DPs in equation (3). We found most of these values by applying equation (5) to MTBF on the net for similar products, except, we assigned 10^{-4} to moving mechanical parts and 10^{-5} to stable parts. We based the value $0.33333(1/3)$, we assigned to lithium-ion batteries, on a net article [9] claiming 2 to 3 years of life for lithium-ion batteries. Table 1 shows these AFR values without solid references in gray background.

Table 1. AFR values of DPs

Design Parameter	AFR
SIM tray	0.00010
SIM	0.00001
Wi-Fi antenna	0.00876
Cellular antenna	0.00876
Taptic engine	0.00548
Digitizer	0.00833
LCD screen	0.01667
Bottom microphone	0.01667
Ear speaker	0.00024
Bottom speaker	0.00024
Front camera	0.00253
Front microphone	0.01667
Rear microphone	0.01667
Regular camera	0.00253
Wide-angle camera	0.00253
Flashlight	0.00012
PC board	0.00001
Power button	0.00006
Silent mode button	0.00006
Volume up button	0.00006
Volume down button	0.00006
Lightning jack	0.00010
Induction coil	0.00833
Li-ion Battery	0.33333
Cables	0.00001
Gaskets	0.00001
Case	0.00001

Other lithium-ion battery articles exist with different life spans up to 6 years. One of the authors, Iino, has been using an iPhone 11 with the first photograph taken in November, 2016, thus, he has been using this unit for over 6 years with the original battery it came with. He also has an iPhone 5 in use since 2013. It only served as a phone unit for about two years and then has been left untouched until recently for looking up meaning of words at night. The original battery for this unit is also in service.

The AFR for smartphone parts largely depend on how the owner use the phone. Most academic users communicate via email on their PCs. Both authors of this paper receive hundreds of mails and reply to about 10% of them. Communications on the smartphone are, for both authors, twenty or less a day. Iino looked up his average screen time with his current iPhone 11 which showed 28 minutes. This is largely less than 4 hours and 23 minutes for an average US mobile user [10]. We calculated the AFR for liquid

crystal display with the 4.5 hours of average screen time, and 100,000 hours of life [11] that gives an MTBF of $100,000 / (4.5 \cdot 365) = 60$ years, and 0.01667 is $1/60$ from equation (3).

Another note is that we did not include the more common problem of causing damage to smartphones, that is, dropping them on ground or on floor [12].

3.2 Functional significance and its failure rate

With the AFR estimates in the previous section, we can calculate annual failure rate of each FR (FFR) with:

$$FFR_i = \sum_{j=1}^m E_{ij} \cdot AFR_j \quad (6)$$

where FFR_i is the functional failure rate of the i -th FR, E_{ij} is DM element of the i -th row and j -th column and AFR_j is the AFR of the j -th DP.

Table 2 shows the resulting calculation results in the yellow column. Note that we also divided the results with 9 to keep the results less than 1. This is equivalent to using link strength values of (0, 1/9, 1/3, 1) instead of (0, 1, 3, 9) at the end of Section 2.3. The numbers 1, 3, and 9 are good for assigning quantities that correspond to human feelings, however, in calculating probabilities, keeping them less than 1.0 is more intuitive.

Table 2, in its second column from the left, shows the significance value S_i the authors assigned to each FR, e.g., establishing connection to the internet via Wi-Fi or cellular network is essential to a smartphone, as well as the user commanding the smartphone what to do via the digitizer screen. On the other hand, illuminating the vicinity with the LED mounted on the back side is a convenient function, however, not essential to the smartphone. Here, we used the quantities 1, 3, and 9 again with a larger number assigned to a more important function. The third column from the left in Table 2 is the product of the significance of i -th FR (S_i) and its functional failure rate (FFR_i). We call this quantity Significant Functional Failure Rate ($SFFR_i$) expressed in the following equation.

$$SFFR_i = S_i \cdot FFR_i / 9 \quad (7)$$

Note that all the electricity related functions are strongly affected by the lithium-ion battery AFR. We highlighted $SFFR$ values of 0.3 or higher in Table 2 with pink background. They turn out to be smartphone functions that depend on power from the lithium-ion battery with user significance values of 9. Note the division by 9 in equation (7) is, again, normalizing the largest intuitive significance of 9.

Table 2. Significance and Functional Failure Rates of Smartphone FRs

[illegible]

4 Discussion

4.1 Lithium-ion battery life

The 2 to 3 years of life for lithium-ion batteries neglects the fact that they are replaceable upon them reaching the end of their lives. Many users have experience with swollen lithium-ion batteries [13]. Pushing on a bulged lithium battery can lead to smoke or fire, thus, if we feel that our smartphones have grown fatter, they no longer stay flat on a flat surface, or the batteries start to drain quickly, we turn them in for service to have their batteries replaced.

Table 3. Significance and Functional Failure Rates of Smartphone FRs with AFR of Li-ion battery at 0.33333 and at 0.00010

	Significance of Function	SFFR: Significant Functional Failure Rate AFR (Li-ion)= 0.33333	FFR: Functional Failure Rate AFR (Li-ion)= 0.33333	SFFR: Significant Functional Failure Rate AFR (Li-ion)= 0.00010	FFR: Functional Failure Rate AFR (Li-ion)= 0.00010
Allow SIM change	3	0.00003	0.00010	0.00000	0.00010
Provide user ID	3	0.11112	0.33335	0.00000	0.00012
Provide net/cell conn.	9	0.34502	0.34502	0.00045	0.01179
Notify text/call IN	3	0.11479	0.34438	0.00043	0.01115
User inputs intention	9	0.34167	0.34167	0.00032	0.00844
Display information	3	0.11667	0.35001	0.00065	0.01678
Display caller	3	0.11667	0.35001	0.00065	0.01678
Catch voice (phone)	3	0.11667	0.35001	0.00065	0.01678
Output voice	3	0.11122	0.33366	0.00002	0.00043
Catch front picture	3	0.11196	0.33587	0.00010	0.00264
Catch voice (SIRI)	3	0.11667	0.35001	0.00065	0.01678
Capture sound	3	0.11667	0.35001	0.00065	0.01678
Capture picture	3	0.11224	0.33671	0.00013	0.00348
Illuminate vicinity	1	0.03705	0.33346	0.00001	0.00023
Execute algorithm	9	0.33335	0.33335	0.00000	0.00012
Turn unit ON/OFF	9	0.33340	0.33340	0.00001	0.00017
Force silent mode	3	0.11113	0.33340	0.00001	0.00017
Force Vol UP	3	0.11113	0.33340	0.00001	0.00017
Force Vol DOWN	3	0.11113	0.33340	0.00001	0.00017
Interface headset	3	0.11115	0.33344	0.00001	0.00021
Data transmission	3	0.11115	0.33344	0.00001	0.00021
Allow power charge	9	0.33615	0.33615	0.00011	0.00292
Provide power	9	0.33334	0.33334	0.00000	0.00012
Route electricity	9	0.33334	0.33334	0.00000	0.00011
Block moisture	3	0.00000	0.00001	0.00000	0.00002
Hold parts together	9	0.00001	0.00001	0.00000	0.00001
Define books	3	0.00186	0.00557	0.00000	0.00557

If the user replaces a swollen battery, the life is elongated to 6 years from 3 years, and repeating this replacement can keep the life of the battery function of supplying

power indefinite. Table 3 in its right two columns shows the SFFR and FFR when the AFR of lithium-ion battery is set to 0.00010, a value that assumes timely battery replacement by the user. Note that some of the FRs with significance of 9 gain high reliability, e.g., “Executing algorithm” or “Turning unit ON/OFF.” The FRs of “Providing net/cell connection” and “User inputs intention” with significance 9 keep relatively high SFFR among the FRs due to their respective reliance on “Cellular antenna” and “Digitizer.” When AFR of the lithium-ion battery drops low with the user intervention, AFR values of other DPs of antennas, digitizer, and microphones turn relatively high.

5 Conclusion

Risk assessment of the type shown in Fig. 1 has a set of events and consequences that are identical and, thus, assumes each consequence depends only on its corresponding event and is independent from all other events. Such risk assessment, therefore, may be misleading, however, practicing one is far better than not performing any risk analysis.

The graphical interface of a DRG makes the zig-zagging easier in trying to figure out an uncoupled set of FRs for a design.

When armed with an algebraic tool of matrix computation, like AD, we can carry out risk analysis in a more systematic way. In the case of a smartphone, the power source of lithium-ion battery and cables that route the power and electrical signals influence almost all functions of the unit and without them, a smartphone turns into just a shiny block.

In our case of smartphone analysis, we identified its weak point of lithium-ion battery, however, with the user deciding to replace an old battery, severity of unacceptable risks drop.

Upon analyzing risks associated with more catastrophic events like earthquakes, tsunami, or tornadoes, we cannot intervene with the events themselves and thus, all we can do to reduce severity of consequences is to work on elements of the matrix, i.e., the link strength of 1, 3, and 9 and make the links weaker. For example, if we have a building that is very likely to collapse with a magnitude 7 earthquake (a strong link 9), we can reinforce its structure, so it is unlikely to collapse in case of such an event (a weak link 1).

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