

Axiomatic Design using Multi-Criteria Decision Making for Material Selection in Mechanical Design: Application in Different Scenarios

Alessandro Giorgetti¹[0000-0002-8232-1523], Fernando Rolli²[0000-0003-0499-4455], Vincenzo La Battaglia²[0000-0002-5018-3314], Stefano Marini² and Gabriele Arcidiacono²[0000-0002-7712-1009]

¹ Department of Industrial, Electronic and Mechanical Engineering, Roma Tre University, Rome 00146, Italy

² Department of Engineering Science, Guglielmo Marconi University, Rome 00193, Italy
alessandro.giorgetti@uniroma3.it

Abstract. In mechanical design, the selection of material alternatives has become a pressing issue due to the progressive growth in the complexity of mechanical systems in search of a continuous increase in performance and the presence of a wide range of possible materials. Moreover, there are many requests for projects, which makes the choice of material a decisive activity for the success or failure of the project itself. Multi-criteria decision-making (MCDM) describes the systematic approaches developed to evaluate alternatives in terms of multiple and often conflicting objectives and identify the cluster of optimal choices. Several methodological approaches combining axiomatic design and MCDM methods have been proposed in the literature. However, it is only in recent years that this methodological combination has found fertile ground as a decision-support tool with interesting applications in the field of material selection. This paper aims to analyze the current state of the art in integrating of axiomatic design and MCDM methods considering different scenarios of the available information in material selection in the field of mechanical design.

Keywords: Material Selection, Multi-Criteria Decision Making, Axiomatic Design.

1 Introduction

1.1 Materials selection problem

Material selection has been considered one of the critical elements of sustainable development as the process motivates the selection of materials that aid in following cleaner production, saving resources and energy, and bringing economic efficiency to any manufacturing enterprise. Materials have a key role throughout the manufacturing as well as the design process. However, selecting the best possible material alternative is a challenging task [1-3] owing to the increasing availability of a large number of materials [4, 5]. During the selection process, many attributes of the materials need to be considered, e.g. the mechanical properties, physical properties, thermal properties, magnetic properties, wear, oxidation, and corrosion behavior. Moreover, a sustainable lifestyle has become necessary due to environmental constraints [4]. Therefore, sustainability adds another criterion that should be considered when selecting a suitable material. In short, the material selection process is a multiple-criteria decision-making

problem. In order to achieve the best solution, the researchers have proposed different procedural steps to arrive at the optimal decisions for material selection strategy [4, 5]. The findings suggest the following stages of a typical material selection process: 1) creating a group of alternative solutions based on the performance requirements, which constitute the selection criteria; 2) screening of the initial solution; 3) ranking and comparing the set of alternatives and 4) identifying an optimal solution. The findings of the past research have enunciated that regardless of the relation between material and process selection, the two main critical aspects for an appropriate material selection are screening and ranking [4].

1.2 Scope

The introduction of axiomatic design as a design methodology in the industrial field is based on two axioms [6]:

- The independence axiom consists of maintaining the independence of functional requirements (FRs), where FRs are defined as the minimum number of independent requirements that characterize the project objectives.

- The information axiom allows us to select the least complex design solution from a finite set of independent solutions. This second axiom states that the design with the highest probability of meeting the requirements is the best design choice, i.e., the one with the least information content.

Several methodological approaches have been proposed in the literature that combines axiomatic design and multi-criteria decision making (MCDM) methods [7-17]. However, only in recent years, this methodological combination has found fertile space as a decision-making tool with interesting applications in the field of material selection [7, 8]. The present paper aim is to analyze how these two methodologies can be integrated in this context.

The proposed methodologies use the concept of information content as a discriminator in the selection of mutually alternative solutions. Then, each approach follows an independent development, as authors often re-interpret the information axiom based on MCDM methodologies. We believe no comprehensive theoretical study still defines the conditions of applying axiomatic design as a material selection tool in mechanical design. The risk is that the full potential of axiomatic design will not be exploited or, even worse, only formally optimal solutions will be obtained. Therefore, it is necessary to present a complete and comprehensive overview of how axiomatic design can be used as a decision-making tool in material selection. This goal can be pursued by first defining scenarios for applying the method. Based on each of them, specific conditions of applicability can be detected. In this study, we have identified three basic scenarios in the highest possible generalization. The first scenario corresponds to a situation of complete information on material selection criteria, which coincide with the functional requirements of the material to be selected. The second scenario, on the other hand, is more complex and corresponds to incomplete information on selection criteria corresponding to the functional requirements of the problem. The third and last scenario analyzed is called the partial information scenario. It corresponds to the partial

correspondence between selection criteria and functional requirements. Some selection criteria are nonfunctional requirements (NFRs).

2 Multi-attribute selection: methodological background

MCDM methods are methodologies for selecting a solution from a set of different possible alternatives on the basis of a set of criteria, which may even be conflicting [5]. These methods can essentially be divided into two categories. Multiple objective decision making (MODM) and multiple attribute decision making (MADM). The main difference between the two approaches is that MODM methods perform comparison analysis on a very large set of solutions, potentially even in infinite numbers. In contrast, MADM methodologies aim to select the best solution from a predefined and limited number of alternatives [5]. Usually, MODM methods are based on decision variables that are continuous functions or integers, whereas in MADM methods, the decision variables are discrete values. In this paper, we refer only to MADM approaches because they are directly compatible with the axiomatic design framework. This compatibility stems from the fact that both methodological approaches perform comparative evaluations on a finite set of alternatives. However, at the same time, this finding presents us with the first significant difference between the two methodologies. While axiomatic design allows the generation of alternative solutions based on the application of the independence axiom, MADM methods do not provide any rational mechanism for pre-selecting alternatives (A_i) to be candidates for final evaluation. This pre-selection consists of formal verification of the candidate material's compliance with the properties it is to possess and the simultaneous exclusion of any incompatibilities. This reduces the number of materials to be submitted for final evaluation, facilitating the selective process. In subsection 1.1, we have introduced that the materials selection process consists of 4 stages. Whereas axiomatic design performs the entire four planned stages, from the identification of materials to be evaluated to the selection of the robust product, MADM methods are designed to perform only the last two stages, i.e., the comparison activities and the determination of the best solution. The last two steps are accomplished by constructing an appropriate dimension $n \times m$ matrix called the decision matrix [4] (Table 1). This matrix turns out to be characterized by four essential elements:

- n rows corresponding to the finite set of materials (A_i) subject to selection;
- m columns representing the selection criteria, which in the terminology of MADM methods are called *attributes* (C_j);
- m weighting coefficients (W_j) defining the relative importance of the selection criteria, where $\sum_{j=1}^m W_j = 1$;
- $n \times m$ elements a_{ij} internal to the decision matrix that constitute the evaluation attributed to alternative A_i with respect to the evaluation criterion C_j .

Table 1. Decision matrix

	C_1 (W_1)	C_2 (W_1)	- (-)	- (-)	C_m (W_m)	Score
A_1	a_{11}	a_{12}	-	-	a_{1m}	$\sum_{j=1}^m W_j a_{1j}$
A_2	a_{21}	a_{22}	-	-	a_{2m}	$\sum_{j=1}^m W_j a_{2j}$
-	-	-	-	-	-	-
-	-	-	-	-	-	-
A_n	a_{n1}	a_{n2}	-	-	a_{nm}	$\sum_{j=1}^m W_j a_{nj}$

The decision matrix, as formulated in Table 1, provides a deterministic solution to the material selection problem, although some evaluation criteria may conflict with each other. This solution consists of providing an ordering to the predefined set of alternatives (A_i) based on the weighting coefficients (W_j) [4, 5]. Nevertheless, to achieve this, we have to resort to a process called normalization, representing a specific material selection problem in a corresponding decision matrix. This process varies depending on the particular MADM technique being adopted. In general, it aims to make a_{ij} evaluation elements comparable and define the weighting coefficients of the selection criteria. In this regard, we must consider that the comparison criteria can be heterogeneous. They may be the physical, chemical and mechanical properties of materials, but also economic considerations, environmental sustainability assessments, or cultural and aesthetic aspects. Therefore, MADM methods make comparisons of a multidimensional nature [18], the final results of which may not coincide, as each method has its own particular specificities. In fact, each method proposes a different model for representing selection preferences.

3 Methodological compatibilities

In this section, we analyze under what conditions axiomatic design can be a viable alternative to MADM methods, in what cases, on the contrary, the two approaches can be combined, and finally, what are the conditions of incompatibility. Before continuing the discussion, let us assume that the identification of candidate materials for final selection is always made through the formal application of axiomatic design. In this way, as anticipated in Section 2, a restricted set of materials is pre-selected based on a formal verification of the characteristics that the mechanical component to be designed must possess. Axiomatic design allows these characteristics to be translated into terms of neutral functional requirements, which constitute the criteria for the final selection. Based on this preliminary hypothesis, we can introduce at least three different operational scenarios.

Scenario 1. We can define a *complete information scenario* as a material selection problem for which the evaluation criteria are exclusively the functional requirements that led to the identification of a predefined set of alternatives. In addition, we know the quantitative data needed to apply the information axiom.

Scenario 2. We can define an *incomplete information scenario* as a material selection problem for which the evaluation criteria continue to be the functional requirements, but we do not have all the quantitative data needed to apply the information axiom. Some criteria may have an evaluation in qualitative terms based on subjective expert judgments.

Scenario 3. We can define a *partial information scenario* as a material selection problem for which the evaluation criteria are only partly the functional requirements, which have guided activities to identify the set of candidate materials. In this case, the final selection also takes place based on nonfunctional criteria.

3.1 Materials selection under conditions of complete information

In a complete-information scenario, applying the information axiom allows us to identify the most suitable material for our objective. In this case, the accuracy of selection is related to the designer's ability to exhaustively represent the specifications of the material to be selected in terms of functional requirements and design constraints. Axiomatic design can directly carry out all four steps involved in the selection process. It is not necessary to implement a normalization process to obtain a decision matrix such as the one introduced in section 2 [19]. Therefore, the application of the information axiom is an alternative tool to traditional MADM methods in selecting a material based on a finite set of candidates. The application of this axiom is to identify the material with the least information content [20]. By definition, the information content associated with a specific functional requirement FR_i is defined as follows:

$$I_i = \log \left(\frac{1}{P_i} \right) \quad (1)$$

In this case, P_i is the probability that the material under evaluation meets the i -th functional requirement. To extend this concept to a complete system, we have to resort to the mathematical properties of logarithms and algebraic properties of square matrices. Preliminary application of the independence axiom allows us to submit candidate materials to a functional verification, which, in essence, establishes the existence of the requirements and the absence of incompatibilities. This preliminary verification allows us to represent the mapping between the problem's intended functional requirements (FR_i) and candidate material properties (DP_i) in terms of a design matrix (Fig. 1).

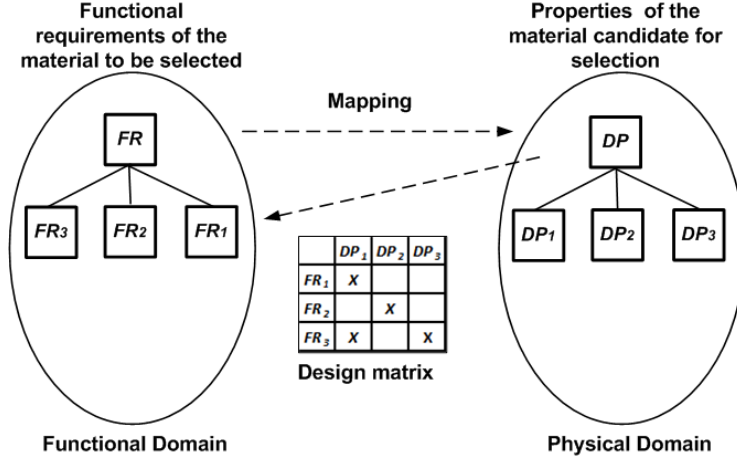


Fig. 1. Axiomatic design as a material selection tool for mechanical component design

In axiomatic design, design matrices that meet the independence axiom can only be diagonal (uncoupled) or triangular (decoupled). For diagonal matrices, the total information content (I_{tot}) is equal to the sum of the information content of all functional requirements (I_i) since they are, by definition, independent [19].

$$I_{tot} = \sum_{i=1}^n I_i = \sum_{i=1}^n \log\left(\frac{1}{P_i}\right) = -\log\left(\prod_{i=1}^n P_i\right) \quad (2)$$

Instead, there are functional coupling situations for decoupled representations [21]. In these cases, the FR_{i+1} requirement depends on the occurrence of the FR_i requirement [22]. This means that the probability that the FR_{i+1} requirement is satisfied by the DP_{i+1} property of a given candidate material is conditional on the probability that the previous FR_i requirement is satisfied by the DP_i property of the same material. However, it is always possible to identify a sequence of conditional probabilities that guarantee functional independence in the corresponding design matrix. Therefore, eq. 2 is also valid in the decoupled matrix case, albeit using conditional probabilities in the functional coupling relations [19]. Under these conditions, axiomatic design allows the selection of a robust material with respect to the functional requirements that have been formalized. Unfortunately, in several cases, the application of the information axiom in its standard formulation has limitations. First, in complex selection problems, for example consisting of the presence of many functional requirements, the use of the information axiom can be complicated [21]. For this reason, there is sometimes a tendency to replace the application of the information axiom with the adoption of MADM methods. As we saw in section 2, these methods are designed to allow us to provide a finite set ordering of alternative solutions.

Nevertheless, the solution identified may not be robust because the use of weighting coefficients determines functional dependence among the selection criteria [19]. Second, the available data may not be quantitative or there may be numerous nonfunctional aspects to consider. In these cases, the information axiom can no longer be applied in its standard approach. The following subsections elaborate on these situations.

3.2 Materials selection under conditions of incomplete information

In an incomplete information scenario, some criteria admit only subjective judgments. For example, in mechanical design, sometimes we can only provide a subjective assessment based on linguistic terms (low, medium, high) to assess the corrosion level of a material. In these cases, incomplete information depends on the vagueness of attributable judgments [4, 5]. Therefore, many authors have proposed the use of fuzzy theory so that the information axiom can be applied to numerical data [7, 12].

Fuzzy approach. Fuzzy AD (FAD) methodology is based on conventional axiomatic design. However, crisp ranges are replaced by fuzzy numbers representing linguistic terms (Fig. 2). In Figure 2, triangular fuzzy numbers (TFNs) are shown. The intersection of TFNs representing design and system ranges presents the common area [23-27]. Firstly, the information content is calculated in a non-fuzzy environment. Then information content in a fuzzy environment is calculated as follows:

- $I_i = \infty$ if the intersection between two adjacent triangles is an empty set;
- $I_i = \log \left(\frac{\text{Area of system range}}{\text{common area}} \right)$ if, on the other hand, the common area is not an empty set.

Even in this case, the best solution is the one with the least information content.

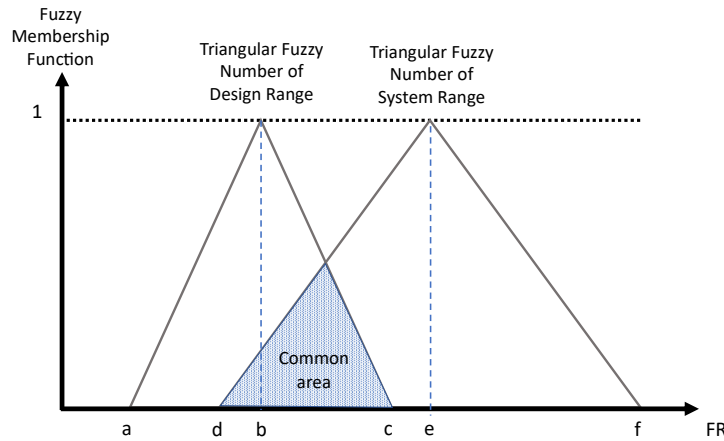


Fig. 2. System-design ranges and common area in fuzzy environment.

Fuzzy axiomatic design presents two fundamental limitations. First, this methodology relies on expert judgment to determine the degree of fuzziness in the design parameters. This can introduce subjectivity into the design process, which may lead to inconsistencies or biases in the design. Recently, advanced approaches derived from fuzzy theory have been proposed, such as the intuitionistic fuzzy set (IFS) and the neutrosophic (NS) method. In particular, the latter approach seeks to overcome this limitation of fuzzy theory by introducing evaluations of "truth", "indeterminacy" and "falsity" into the model. Abdel-Basset et al. [13] used this approach in conjunction with axiomatic design in the selection of medical instrumentation. Another approach that has been

proposed in recent years is based on Z-numbers [14, 15]. In all these cases, adopting these methodologies aims to reduce uncertainty in applying axiomatic design in incomplete information application scenarios. Uncertainty in a selection problem can also arise from the risk of adopting a particular material. For example, overheating may result in undesirable effects on some mechanical components. Hafezalkotob et al. report in [28] a real-life material selection case for the construction of gas turbine blades. Temperature variation was considered as a risk factor. In this case, the FAD method was modified to include the risk variable associated with blade overheating. Thus, combining elements of risk and the FAD approach, the Risk Fuzzy Axiomatic Design (RFAD) method is obtained. The information content for the RFAD technique is calculated as follows [29, 30]:

$$I_{ij}^r = \log \left(\frac{1}{p_{ij}(1 - r_{ij})} \right) \quad (3)$$

In this case, r_{ij} is a risk factor with a value in the range of zero and one. Comparing the information contents of FAD and RFAD approaches, it is explicit that each I_{ij}^r is greater than its corresponding I_{ij} . Greater risk factor r_{ij} leads to a higher value of information content I_{ij}^r . In addition, FAD can be a complex and time-consuming process, especially when dealing with systems with many functional requirements or design parameters. In these cases, the information axiom can be reformulated in terms of MADM methods. Generally, this methodological re-interpretation consists of introducing weighting coefficients to be assigned to the selection criteria.

Selection based on weighted attributes. The importance of criteria in decision-making problems is often not similar. Consequently, the relative importance of criteria should be considered to achieve a realistic solution. In general, the significance coefficients can be computed using objective, subjective, or integrated techniques [28]. Subjective significance coefficients are achieved from experts' opinions while objective significance coefficients are obtained using the decision matrix's values without utilizing experts' judgments. The two types of significance coefficients may be combined. Different techniques are borrowed from MADM methods for calculating the significance coefficients of criteria [30-33]. From a methodological point of view, the use of these weighting techniques consists of combining FAD and RFAD approaches with MADM methods. In this paper, we briefly introduce the three main MADM approaches used to determining weighting coefficients in situations where information is incomplete: the information entropy method, the analytic hierarchy process and the best-worst method.

Information Entropy Method. Entropy is based on the classical measures of Boltzmann and the second law of thermodynamics [28, 35]. The idea of entropy in information science, initially suggested by Shannon [35], is a tool for specifying the uncertainty of a variable. The general concept of Shannon's entropy is to evaluate the significance coefficient of each criterion from the distribution of data over variables. The Shannon entropy has been utilized with combinations of many MADM techniques for various applications in material selection problems [4, 5, 28, 32, 34]. Hafezalkotob et al. [32] developed the RFAD method with the integrated Shannon entropy significance

coefficients to generate an entropy-weighted risk-based fuzzy axiomatic design (WRFAD) approach. The information content of the WRFAD technique is based on the integrated Shannon significance coefficients. However, this approach has a fundamental disadvantage. It requires a significant amount of input data, which can sometimes be difficult to obtain. The accuracy of the results depends on the quality of the data, and inaccurate or incomplete data can lead to erroneous decisions.

Analytic Hierarchy Process. The analytic hierarchy process (AHP) is a decision support method developed to complete problems by breaking the solution problems, grouping them, and arranging them into a hierarchical structure [4, 5]. This method uses a comparison of criteria paired with a measurement scale that has been determined to obtain priority criteria. The main input of the AHP method is experts' perception, so there is a factor of subjectivity in retrieval decisions [36-38]. This aspect is both a strength and a weakness of this method. It is a strength of the method because it is a powerful tool for modeling complex decision-making situations. However, considerable uncertainty and doubts in the evaluation affect the accuracy of the data and results obtained. Based on this consideration, another theory was developed, namely Fuzzy 40

Analytic Hierarchy Process. The fuzzy AHP is a method of AHP developed with fuzzy logic theory [11, 39]. The fuzzy AHP method is used similarly to the method of AHP. It is just that the fuzzy AHP method sets the AHP scale into the fuzzy triangle scale to be accessed priority.

Best-Worst Method. Rezaei [33] developed the best-worst method (BWM) based on a consistency comparison system. The method has been further extended by Guo and Zhao [40] by integrating the fuzzy set data into the approach (called fuzzy BWM or FBWM). In BWM, the pairwise comparison between the best and worst criteria is defined as a reference comparison in which the best and worst criteria are computed. Additionally, the secondary comparison occurs when neither of the selected criteria is defined as the best or the worst element. In real-world problems where uncertainty and ambiguity of decision-maker exist, it is tough to evaluate the accurate weights of the criteria. Based on the BWM approach, the hybrid hierarchical best-worst fuzzy axiomatic design (HB-WFAD) selection method, which combines axiomatic design and FBWM, has been proposed [39]. In this case, criteria weights are calculated by exploiting the BWM technique, which has the advantage of requiring a limited amount of information. However, subjective value judgments always remain a critical issue in the selection process.

3.3 Materials selection under conditions of partial information

In some situations, selection procedures may involve elements that difficultly can be formalized in terms of functional requirements. Design constraints most often belong to this category. Design constraints are limitations to design, which may depend on the material's physical, chemical, or mechanical properties to be selected or on economic reasons, product availability, environmental and social sustainability of the production process, or even aesthetic and cultural motivations [4, 41-45]. In terms of axiomatic

design, these constraints can be classified into two categories [46]: input constraints if they relate to the fulfillment of a specific functional requirement that a mechanical component must possess, for example, a certain threshold of maximum allowable heat transmittance. In contrast, system constraints do not relate to a specific material property. For example, materials from different countries with the same properties may have been produced through processes with different environmental and social impacts. Many companies require their suppliers to ensure high levels of environmental and social sustainability [29]. Axiomatic design does not provide formal rules for treating these elements, as is the case with functional requirements. To overcome this limitation, Mabrok et al. [37] proposed to consider these elements as nonfunctional requirements (NFRs) and to replace the functional domain in Figure 2 with a new domain, called the requirement domain, which includes both types of requirements (Fig. 3).

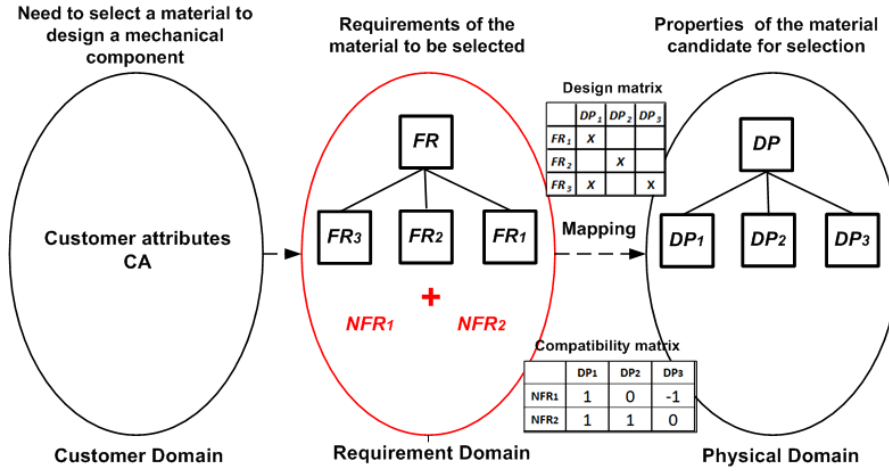


Fig. 3. Selecting materials for mechanical component design by including nonfunctional requirements in axiomatic design.

This intuition allows the formal inclusion of nonfunctional requirements in axiomatic design. At the operational level, this idea can be accomplished through the definition of a new matrix called the extended design matrix (eq.6). This new matrix includes two blocks, the design matrix ($n \times n$) related to the n functional requirements of the selection problem and the compatibility matrix of size $k \times n$ for the k associated nonfunctional requirements. The latter matrix relates the nonfunctional requirements to the properties of the material submitted for verification. In the example shown in Figure 3, the compatibility matrix can be constructed based on three values:

- $a_{ij}=1$ if design parameter j -th satisfies nonfunctional requirement i -th;
- $a_{ij}=0$ if design parameter j -th is indifferent to nonfunctional requirement i -th;
- $a_{ij}=-1$ indicates, on the other hand, that design parameter j -th violates the nonfunctional requirement i -th.

If we refer to the example in Figure 3, the relationship between the requirement and physical domains can be represented by eq.4.

$$\begin{bmatrix} (FR_1) \\ (FR_2) \\ (FR_3) \\ (NFR_1) \\ (NFR_2) \end{bmatrix} = \begin{bmatrix} X & 0 & 0 \\ 0 & X & 0 \\ X & 0 & X \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} DP_1 \\ DP_2 \\ DP_3 \end{bmatrix} \quad (4)$$

In practical terms, the design matrix represents the traditional mapping between functional requirements and selected material characteristics (design parameters). In contrast, the compatibility matrix measures the level of compliance with respect to the nonfunctional requirements provided by the selection. This way, eq.4 introduces two pre-selection operations on candidate materials for final selection. An initial pre-selection is made to identify candidates that meet the functional requirements as in the traditional axiomatic design approach. This pre-selection allows a finite set of candidate materials to be identified. Then, the compatibility matrix allows us to evaluate with respect to this first set of materials, which ones meet the nonfunctional requirements posed by our problem. This second intervention allows us to restrict the set of candidate materials even further for final selection. Finally, we must proceed to the final selection.

On the other hand, as far as the final selection is concerned, we can no longer resort to the information axiom, or at least as defined in Section 2, since we also must consider evaluation criteria that derive from nonfunctional requirements. In this case, the various MADM methods provide a powerful tool for making the final selection [37]. Recently, some studies have proposed the AHP methodology, which has the advantage of grouping the selection criteria hierarchically-mindedly into groups and subgroups [36, 37]. In this sense, the selected material constitutes the best solution with respect to modeling the operational context. However, it may not coincide with the robust solution. Therefore, we may have found a suboptimal solution with respect to the functional requirements due to the simplifications introduced by assuming the nonfunctional requirements and then applying the MADM methods.

4 Conclusions

In mechanical design, the selection of materials to be used is becoming an increasingly complex problem because of the wide availability of alternative materials and the progressive emergence of new constraints. Currently, it is common practise to consider elements and performance related to environmental and social sustainability in the design process, in addition to the usual selection criteria such as the physical, chemical and mechanical properties and the cost of the component. The component must be manufactured to meet specific technical and economic requirements and minimize the social and environmental impacts throughout its entire life cycle, from raw material extraction to end-user use and dismantling. In this context, axiomatic design allows these even mutually conflicting requirements to be articulated in formal terms of functional requirements and design constraints. This specificity of axiomatic design allows these elements to be included as selection criteria in a unified framework. However, the increasing complexity of problems in scenarios with incomplete or partial information

makes the final choice very difficult. Therefore, scholars resort to the extension of the information axiom to simplify its applicability. In chronological order, the use of fuzzy theory was the first step in this process of adapting axiomatic design in complex application contexts. Then, other methodologies were presented that further simplify the information axiom by applying MADM methods (AHP, information entropy, BWM). In this case, a robust solution is given up for suboptimal solutions. The latter has the advantage of providing a solution even to very complex selection problems, unsolvable with the traditional approach. The partial-information scenario is emblematic of this trade-off situation. The presence of several system constraints can make it complicated to formulate the selection problem solely in terms of functional requirements. Instead, the interpretation of system constraints as nonfunctional requirements simplifies the determination of a solution. However, the final choice may not coincide with the robust solution with respect to axiomatic design. It is certainly a better solution than the simplified model based on nonfunctional requirements. In this sense, research is increasingly focused on studying the compatibility between axiomatic design and MADM methods in order to reduce the gap between robust solutions and suboptimal choices.

References

1. Giorgetti A., Cavallini C., Arcidiacono G., Citti P.: A mixed C-Vikor fuzzy approach for material selection during design phase: A case study in valve seats for high performance engine. *International Journal of Applied Engineering Research* 12(12), 3117-3129 (2017).
2. Arcidiacono G.: Development of a FTA versus Parts Count Method Model: Comparative FTA. *Quality and Reliability Engineering International Journal* 19, 411-424 (2003).
3. Ceccanti F., Giorgetti A., Cavallini C., Arcidiacono G., Citti P.: Comparative evaluation of fuzzy axiomatic Design and IAMS comprehensive VIKOR approaches for material selection in mechanical Design. *Journal of Engineering Design and Technology* 13(1), 80-87 (2020).
4. Jahan A., Edwards K., Bahraminasab M.: *Multi-criteria Decision Analysis for Supporting the Selection of Engineering Materials in Product Design*. 2nd edition. Butterworth-Heinemann, Oxford (UK) and Cambridge (USA) (2016).
5. Rao R. V.: *Decision Making in the Manufacturing Environment. Using Graph Theory and Fuzzy Multiple Attribute Decision Making Methods*. 1st edition. Springer-Verlag, Series in Advanced Manufacturing, London (2007).
6. Suh N.P.: *Axiomatic Design - Advances and Applications*. 1st edition. Oxford University Press, New York (2001).
7. Dhivya J., Maheswari K., Saroja M. N.: A Fuzzy Logic Approach for Material Selection Problem. In: *International Conference on Materials Research in Science and Engineering (KMRSE'21)*. AIP Conference Proceedings 2446, Coimbatore, India (2022).
8. Gul M., Celik E., Gumus T.A., Guneri A. F.: A fuzzy logic based PROMETHEE method for material selection problems. *Beni-Suef University Journal of Basic and Applied Sciences* 7, 68-79 (2018).
9. Cicek, K., Celik, M.: Multiple attribute decision making solution to material selection problem based on modified fuzzy axiomatic design-model selection interface algorithm. *Materials and Design* 31(4), 2129-2133 (2010).

10. Candan G., Kir S. and Yazgan H. R.: Solution of material selection problem using fuzzy axiomatic design and DEMATEL methods. *Acta Physica Polonica Series* 131(1), 24-27 (2017).
11. Kahraman C., Cebi S.: A new multi-attribute decision making method: hierarchical fuzzy axiomatic Design. *Expert Systems with Applications* 36(3), 4848-4861 (2009).
12. Maghsoodi A. I., Hafezalkotob A., Azizi I., Maghsoodi S. I., Hafezalkotob A.: Selection of waste lubricant oil regenerative technology using entropy-weighted risk-based fuzzy axiomatic design approach. *Informatica* 29, 41-74 (2018).
13. Abdel-Basset M., Mohamed M., Mostafa N.N., El-Henawy I.M., Abouhawwash M.: New multi-criteria decision-making technique based on neutrosophic axiomatic Design. *Scientific Reports* 12(1), (2022).
14. Aydoğan S., Günay E.E., Akay D., Kremer G.E.O.: Concept design evaluation by using Z-axiomatic Design. *Computers in Industry* 122 (2020).
15. Liu Q., Chen J., Wu Y., Yang K.: Linguistic Z-numbers and cloud model weighted ranking technology and its application in concept evaluation of information axiom. *The Journal of Supercomputing* 78, 6061–6089 (2022).
16. Rolli F., Parretti C., Giorgetti A., Arcidiacono G, Citti P.: A Mixed Axiomatic Design/MADM approach for the sustainability representation of an offshore hydrocarbon extraction facility. *Sustainability*, forthcoming.
17. Arcidiacono G., Berni R., Cantone L., Nikiforova N.D., Placidoli P.: Fast Method to Evaluate Payload Effect on In-Train Forces of Freight Trains. *The Open Transportation Journal* 12, 77-87 (2018).
18. Zheng P., Wang Y., Xu X., Xie S.Q.: A weighted rough set based fuzzy axiomatic design approach for the selection of AM processes. *International Journal of Advanced Manufacturing Technology* 91 (5-8), 1977-1990 (2017).
19. Gonçalves-Coelho, A., Fradinho, J. M. V., Gabriel-Santos, A., Cavique, M., Mourão, A. J. F.: (2022). How to handle the design preferences with Axiomatic Design. *IOP Conference Series: Materials Science and Engineering*, 1235 (2022).
20. Fradinho J., Gonçalves-Coelho, A. M.: The Information Axiom and Robust Solution. In: Suh, N.P, Cavique M., Foley J.T. (eds), *Design Engineering and Science*. Springer, First Edition, 307-325 (2021).
21. Slătineanu L., Dodun O., Coteață M., Dulgheru V., Dușa P., Banciu F., Beșliu I.: Selection of a Solution When Using Axiomatic Design. In: *International Conference on Axiomatic Design ICAD 2017*. MATEC Web of Conferences 127, 01019 (2017).
22. Brown C.A.: Kinds of coupling and approaches to deal with them. In: *Proceedings of 4th ICAD2006, The Fourth 612 International Conference on Axiomatic Design, Firenze* (2006).
23. Vinodh S., Kamala V., Jayakrishna K.: Application of fuzzy axiomatic design methodology for selection of design alternatives. *Journal of Engineering Design and Technology* 13(1), 2-22 (2015).
24. Cicek K., Celik M.: Multiple attribute decision-making solution to material selection problem based on modified fuzzy axiomatic design-model selection interface algorithm. *Materials & Design* 31(4), 2129-2133 (2010).
25. Karatas M.: 2017 Multiattribute decision making using multiperiod probabilistic weighted fuzzy axiomatic Design. *Systems Engineering* 20 (4), 318-334 (2017).
26. Kannan D., Govindan K. Rajendran S.: Fuzzy axiomatic design approach based green supplier selection: a case study from Singapore. *Journal of Cleaner Production* 96, 194-208 (2015).
27. Celik M., Cebi S., Kahraman C., Er I. D.: Application of axiomatic design and TOPSIS methodologies under fuzzy environment for proposing competitive strategies on turkish

- container ports in maritime transportation network. *Expert Systems with Applications* 36(3), 4541-4557 (2009).
28. Hafezalkotob, A., Hafezalkotob, A.: Risk-based material selection process supported on information theory: a case study on industrial gas turbine. *Applied Soft Computing*, 52 (1), 1116-1129 (2017).
 29. Gormus E., Tasci Durak Z.: A novel approach for green supplier selection problem: fuzzy axiomatic Design with risk factors. *Journal of Management and Economics Research*, 19(2), 1-16 (2021).
 30. Kulak O., Goren H. G., Supciller A. A.: A new multi criteria decision making approach for medical imaging systems considering risk factors. *Applied Soft Computing* 35, 931-941 (2015).
 31. Kulak, O., Kahraman, C.: Fuzzy multi-attribute selection among transportation companies using axiomatic Design and analytic hierarchy process. *Information Sciences*, 170(2-4), 191-210 (2005).
 32. Hafezalkotob A., Hafezalkotob A.: Fuzzy entropy-weighted MULTIMOORA method for materials selection. *Journal of Intelligent & Fuzzy Systems*, 31(3), 1211-1226 (2016).
 33. Rezaei J.: Best-worst multi-criteria decision-making method. *Omega* 53, 49-57 (2015).
 34. Zhang G., Marvel S., Truong L., Tanguay R.L., Reif D.M.: Aggregate entropy scoring for quantifying activity across endpoints with irregular correlation structure. *Reproductive Toxicology* 62, 92-99 (2016).
 35. Shannon, C.E.: A mathematical theory of communication. *The Bell System Technical Journal* 27, 379-423 (1948).
 36. Pourabbas E., Parretti C., Rolli F., Pecoraro F.: Entropy-Based Assessment of Nonfunctional Requirements in Axiomatic Design. *IEEE Access* 9, 157831-156845 (2021).
 37. Mabrok M., Ryan M., Efatmaneshnik M.: Integrating Nonfunctional Requirements Into Axiomatic Design Methodology. *IEEE Systems Journal* 11(4), 2204-2214 (2015).
 38. Saaty T.L.: How to make a decision: The analytic hierarchy process. *European Journal of Operational Research* 48 (1), 9-26 (1990).
 39. Maghsoodi A. I., Mosavat M., Hafezalkotob A., Hafezalkotob A.: Hybrid hierarchical fuzzy group decision-making based on information axioms and BWM: Prototype design selection. *Computers & Industrial Engineering* 127, 788-804 (2019).
 40. Guo S., Zhao H.: Fuzzy best-worst multi-criteria decision-making method and its applications. *Knowledge-Based Systems* 121, 1-9 (2017).
 41. Citti P., Giorgetti A., Millefanti U.: Current challenges in material choice for high-performance engine crankshaft. *Procedia Structural Integrity* 8, 486-500, (2018).
 42. Giorgetti A., Monti C., Tognarelli L., Mastromatteo F.: Microstructural evolution of René N4 during high temperature creep and aging. *Results in Physics* 7, 1608-1615. (2017).
 43. Baldi N.; Giorgetti A.; Palladino M.; Giovannetti I.; Arcidiacono G.; Citti P.: Study on the Effect of Inter-Layer Cooling Time on Porosity and Melt Pool in Inconel 718 Components Processed by Laser Powder Bed Fusion. *Materials* 16, 3920. (2023)
 44. Vezzù S., Cavallini C., Rech S., Vedelago E. Giorgetti A.: Development of High Strength, High Thermal Conductivity Cold Sprayed Coatings to Improve Thermal Management in Hybrid Motorcycles. *SAE Int. J. Mater. Manf.* 8(1) (2015).
 45. A Giorgetti, U Millefanti, V La Battaglia, P Citti, Investigations of Fatigue Damage in a Nitriding Low-Carbon Bainitic Steel for High-Performance Crankshaft, *Metals*, 12(12):2052 (2022).
 46. Liu A.: Problem definition, In: Suh, N.P, Cavique M., Foley J.T. (eds), *Design Engineering and Science*, Springer, First Edition, 167-189 (2021).